A New 4DOF Desktop Parallel Manipulation System (D-PMS) – Kinematic Analysis and Performance Evaluation

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Abstract: In this paper, the basic kinematic features of a set of modular multipurpose manipulation units conceived to work as an individual or integrated system inside of a desktop assembly cell environment are analyzed and some of the specific parameters are evaluated. The basic kinematic concept of the system is presented, in which each of the components - fundamental (Mu-0) and/or auxiliary manipulation units (Mu-X) are 4dof Parallel Mechanism family members, having, symmetric and/or nonsymmetric structures, but all with 2dof in-parallel actuators. The general and/or specific features of motion resulted from an analysis carried out on the identification and evaluation of kinematic parameters of fundamental manipulation unit structure and geometry. Mobility, taken into account the, number and place of the pairs, and displacements by the inverse and direct positions problem model are solved on mobile curves of guidance (MCG) equations. The work is as the first step to explore the system capabilities and consist to be a basis for further kinematic and dynamic studies.

Keywords: 4dof parallel mechanisms, kinematic analysis, topology, mobility, inverse/direct positional problem, 2dof actuators, scissor, parallelograms

I. INTRODUCTION

The progresses in science and technology, seen as fundamental researches or product/production improved solutions, for the purpose of maintaining health, creating goods, etc or, only by increasing the every day comfort at work or home, have been always related with the development of advanced systems. In order to reach and to properly work with the constant increasing high level of values and spectrum for their prescribed (and, required) parameters, new machines, device and/or instruments - seen each or together, as a single system must include and to be conceived on innovative ideas.

Their mechanical features (kinematic, dynamic, etc) have to be in detail analyzed before a practical implementation start, and correct identified (and, evaluated) as variable and constant parameters, from the performances’ level point of view.

As was pointed out before, by some authors (e.g. [1], [2], etc), in the precision assembly, an (semi)automated system is required to be able to perform various simple and/or complex motions tasks over the larger work spaces with an increased performance index, including one or more, as high accuracy and speed, bigger payload capacity, etc. There are several specific tasks, which should be fulfilled by the manipulation system - transport, pick-and place, alignment, machining, soldering, gluing, sealing, inspection, etc. Excepting the standard simple operations including the transfer (transport), or simple positioning, which involve one (1) dof, most of them require more than two (2) DOF motion capability. But not always, the maximum, six (6). The manipulation system must be precise and fast enough, easy adapted at various changes related with the volume and/or type of the production/products to be fabricated. In other words, it should be flexible.

These manipulation systems have to be indeed new and innovative to fulfill these requirements.

They would be true (mobile) robotic systems adapted to a specific application. This will assure not only to attain, a high level of production, in automated processes, but to maintain it in (semi)automated processes helping an operator (human being) by assisting him. (And, by this to be less stressed.) A challenging tasks today, for the precision manipulation, is to conceive and design these (flexible) systems.

As it was revealed from the beginning of this work [3], a gap have existed and a lack of work between micro and mini domain of manipulating small precision components at the desktop level (desktop precision devices, desktop cell, desktop factory, etc). We tried to fill it and proposed some new and/or improved solutions (e.g. [4], [5], etc).

Serial stacked type based large manipulators (robots) even take in to account the last improvements regarding the direct drive solution for actuation or closed loops [6] implementations in their kinematic chain (parallelograms, for example) have proved to be still limited in performances in the terms of speed, stiffness, or accuracy, especially when we spoke about all together. In contrast, they have a good mobility and can fulfill complicated task (including large orientations) involving more the one dof. Most of the smaller size precision positioning/automation units (only, as example [7]) are build also on the concept of staked one axis actuated stages (X, Y, Z, goniometric, etc). The technology of producing and assembly them in standard units is well developed now, making them attractive as price. They have a small space, but good or high accuracy, depending of the design - actuation and guidance, solutions, but becomes bulky when more than 3 dof are required, including the orientation. In contrast, the X, Y, Z robotic axis (units) as for example pneumatic actuated can reach the target with high speed capabilities over large workspaces but with lower accuracy, because of intrinsic inertia. Closed loop based type mechanisms, as in-parallel actuated ones, have still enough resources and potential to substitute the serial mechanisms in accomplishing the task related with the problems from above.

Parallel mechanisms (PM) as closed loop mechanisms having a mobile body (platform) in connection with fixed one (base) by a least two independent (serial) kinematics chains (legs/arms), each of them being actuated by simple actuators [8]. If the number of chains is strict equal with that of dof for the platform, PM is defined as fully parallel mechanism [9]. Applications can be seen now in almost every domain from

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simulators, machine tools, robot manipulators, precision alignment, medicine, etc. where the stiffness, accuracy and dynamics are the primordial features. Their number is difficult to be counted systematized and analyzed, but several attempts were already done.

At the University of Tokyo, Department of Precision Engineering, Takamasu Laboratory, parallel mechanisms started to be studied since several years ago [10] - Coordinate Measuring Machine (CMM), Micro Machine Tools, other precision parallel positioning devices, etc.

Based on the results included in the work [11] a strong collaborations with Japanese companies started (especially, in relation with first produced/exhibited then (1999) high precision PLANAR STAGE [12]). And, as a result, parallel mechanisms with planar 2dof actuation were intensively exploited further, based on these opportunities, by an additional imposed design requirement - sliders to be around as much as possible smaller, in order to work as precision devices for desktop applications. All these actions concretized during the time in several experimental parallel positioning systems and components with 2dof, 4dof and 6dof for the first time conceived and build in Japan [13]. They were proposed, to work in different “scenarios”, as precision manufacturing (assembly) cell, and other enhanced desktop applications caring out different tasks in a high controlled environment [14].

The kinematic synthesis and general design concept of such system as precision modular multi degree of freedom positioning system (MDOF-PS) was presented in [15]. The system’s components are members of PMs class with 2dof actuators.

The work presented in this paper, is a continuation of what was started from the beginning, and an extension of what was recently done. It deals with 4dof PMs positioning family only, and consists in defining more precise the possible manipulation tasks, to be accomplished by the system in a cell, and a kinematic study of its specific features, which are now more in detail analyzed and discussed. This will facilitate not only a better understanding of the system particular behavior, and by this to discover its special characteristics, but help the designers to have an entire set of motion capabilities. In this respect, a global overview of whole mechanism’s working potential is envisaged. Actual analysis is done on a fundamental topological structure and its geometric/kinematic model.

Before this analysis start, a short incursion is proposed in the 4dof PMs achievements from the technical and scientific literature.

II. 4DOF PARALLEL MECHANISMS - KINEMATICS OVERVIEW

In the last few years, a general increased interest can be seen towards the developing of new structures and with improved capabilities [8]. The proposed number is steadily increasing.

Especially, spatial parallel manipulators with fewer degrees of motion than six, but more than three, attracted the attention of both, the researchers and users. They promised not only an advantage of simpler architecture and total low cost, but could provide the best suit for some specific applications taken into account one or more of the features as: number of the dof, type of motion, workspace size, mobility, accuracy, stiffness, etc.

The idea to find out spatial PM structures with fewer (lower) dof (mobility) started from the beginning of the days of PMs.

But, a systematically method in order to analysis and to synthesizing them, in the scientific literature appeared only, recently. Nevertheless, already good steps and results have been done till now, and an increased interest is arising.

As was noted from others authors, the 4dof PM appeared relative early. A general PMs survey is shown on Internet [16] and [17] is periodically updated, and a short analysis was included in [8], too. Other usefuly sources are specialized seminar, symposiums, workshops, etc as PKM, PKS, ARK, etc.

LIRMM (France) laboratory team conceived and continuously improved by design during the years, a family of 4dof PMs (3T+1R) targeting the high speed pick-and-place handling applications as a contra-candidate of SCARA success based on the DELTA one. H4 L is the evolution of H4 concept [18]. A derived family I4 [19], as a fully parallel mechanisms demonstrator with linear actuators (HL) inside of Mach21 European project, a release (HR) loosing the symmetry but with an improved simplicity, and a combination (Par¼) of the first basic two (H4+I4). The actuators four points were proposed as different position of lines (horizontal and vertical) and types (linear or rotative) on the base and the spatial parallelograms still in the position of an important components. The target to have large angular displacement (rotations) was achieved, but the contact points with the platform (nacelle) is divided in two on two moving parts relative to each other, who limited the stiffness and their practical use till now, despite of different design solutions proposed. Other (redundant) structures designed there as Archi double scissor mechanism trying for escaping the singularities of planar parallel mechanisms [20].

For high speed handling applications, two types of DELTA modified structures was conceived at EPFL (Switzerland)– MANTA and KANUK [21]. Depending of how one of the three actuation lines is coinciding or not with others or is perpendicular on. In the first case, three (3) linear actuators are horizontal, one (1) rotational being carried by other. The last case, includes two subcases a lines coincide with other two and/or is perpendicular. There are (4) points of actuation, and the same three chains (2 Pa+1bar) for contacting the platform (3 points). Acceleration (limited only by the value of commercial existent motors) of up to 5g was reported in experimental environment, as the mobile components were light (rods) and the mobile platform sometimes only a bar.

A more sophisticated parallel kinematics architecture was conceived for high-speed machine tools - HITA SST [22], consisting from a tilted (45°) platform caring four (4) linear actuators and parallelograms bars which permit more large angles for the tool (-15° to 115°). The stiffness is a main advantages, but the singularities and intersections of components are not negligible.

A family of two PM with four degrees of freedom having a prismatic additional (1) leg to the (3) three 5R chains, and/or symmetric (4) four 5R chains derived from a spherical structure, was proposed and analyzed from the kinematic point of view in [23] to be more adequate for simulation purposes (ample rotations and high translation).

A consistent work have been done by Liu who intensively studied the unique role of planar parallelograms as PM components [24], [25], etc resulting in proposing some new
parallel designed mechanisms. Simple or with redundant constraints was pointed out the Pa units main advantages and drawbacks. Four dof structures was included, too. In conjunction, first prototype of five axis Chinese Parallel Machine Tool (PMT) with a simple kinematics (2+1) which uses for the tool manipulation at the beginning 3dof planar parallel mechanism (scissor+additional link) and now in use an only improved 2dof, was studied.

An investigative work was carried out on the type synthesis of PM in [26]. Several classifications and new types of composite pairs and sub-chains (limbs) were proposed and after that used for the design of 2-,3-,4- and 5-DOF robotic mechanisms. It was highlighted once more again that composite joints as planar parallelograms (U^) are very useful to design. Several in four points actuated and supported PMs examples were included.

A double four-rod spatial parallel mechanism was from kinematic point of view studied for high speed traveling applications, as machine tool by K. Zhou [27]. It has four linear actuators (sliders) moving along of two straight-lines disposed in parallel from a horizontal plan. Two of the actuators being collinear, the kinematic features are accordingly. The basic results of the kinematics - workspace, direct and inverse positions problem and singularity loci were obtained, for this fully symmetric PM structure. Similar structure, but vertical based was studied from the kinematic point of view (inverse, direct and singularities) in [28]. Three and four degrees of freedom overconstrained parallel manipulators were geometrically analyzed using equivalent screw groups analytical method based on linear independence and reciprocity of screw systems in [29]. As a result the constraints characteristics of a 4dof PM (4-UPU) and three parallelograms units (4R, 4U and 4S) were revealed.

Several new fully-symmetric 4dof PM with three (3RRR(RR), 3RRR(RR), RR(RR))R and four limbs (4-UPU, 4RPR(RR), 4RRR(RR)) have been presented as a result of the simple and easy constrained-synthesis process of lower mobility PM in [30] by considering mobility, properties and the pair combination of limbs.

A humanoid shoulder complex motion was simulated by the particular design of a three dof traditional mechanism (U(P)U) with an aditional central leg ((P)S) in [31]. Other (nonsymmetrical) and non-fully 4dof structures, as for example with 2dof actuator in one leg are [8].

From the above short analysis, we can conclude that,

- The result of last actions was an increased number of theoretically proposed structures with less or more practically impact and consequences,
- A few structures/architectures able to perform various (pre)defined tasks required in industry, science or as a demonstration purpose only, is effectively working as models (prototypes), and not exciting devices, as tremendous practical implementations exist,
- Their kinematic (and/or, dynamic) analysis and maximum values evaluation of performances obtained in the experiments (prototype), by identifying and specifying their specific features, is a normal process, paving the way towards their fast further implementation,
- Spatial parallelograms remain one of the main components in their mechanisms design. Unfortunately,
- most of the structures are fully (symmetric) types and as a result with four points actuation (and, at least same number for platform) contact, using only one actuator in a chain, who give give the impression of complex and large foot bed space occupied,
- there are no strong relation or connection with the scissor and (planar) parallelograms mechanisms included in their chains. We already took in to the consideration these important aspects and proposed some structure which encompass with the previous drawbacks, but maintain the already advantageous solutions proved from the particular or general design, combining them in own new ideas adapted to the specific desktop application. The result is a new concept of innovative symmetric PMs solution based on a simplified (concentrated) actuation contact points and particular arrangement of the pairs in only two chains, who provide a more flexibility to work in large workspaces, and as a structure with the best balance advantages/ drawbacks. It is bellow explained in relation with an desktop assembly applications environment.
III. 4DOF DESKTOP PARALLEL MANIPULATION SYSTEM (D-PMS) KINEMATICS CONCEPT

The 6DOF Symmetric Parallel Mechanisms (SPM) class with two actuated pairs on a kinematic chain (Kc), each with one degree of freedom (2x1dof) concentrating the actuation in only three (3) points have been proved to have some advantages comparing with the traditional ones, having only one degrees of freedom (6x1dof). During the time several common and new architectures with 4dof and 6dof were proposed and conceptual designed [15]. Their general aspects promised an increased suitability for various simple or complex tasks inside of the assembly cell involving in the same time high speed and accuracy, and/or stiffness for caring heavy objects (weights), working under variable and appreciable forces (loads).

The 4dof robotic set of units working inside of an assembly cell in order to assist an operator (human and/or robot) to perform the required tasks, in time and with the imposed pre-established succession of operations could be seen as an individual (separated), but integrated (sub)system and is shown in Fig. 1. It consists mechanically, from several components with multidegrees of freedom (MDOF) capabilities – manipulation units (Mu0-MuX), each of them (or, in cooperation) caring out specific tasks (and, motions) derived from the technological plan: machining, press, quality control/inspection, transportation, etc, and with the required kinematic and dynamic set of parameters - velocities, accelerations, force, etc. The tasks are to be fulfilled inside of the general permitted system’s accuracy given by each unit.

The positioning system of each Mu was centered on a 2dof independent positioning unit module, as parallel planar stages (XY), planar motors [13], or another innovative ones solution (self-propelled [32]), who serves as actuator and the module for the entire family. From the kinematic design point of view, each of the components of the family was derived from a development process taken in to account the transformation of motion mechanism – deformable triangle (half scissor), as Fig.1 shows (which is seen as (PP)RR, a fundamental parallel structure, too).

Fig. 2 2DOF parallel manipulation structures-a) ordinary (line) and - b) anti (dashed) paralelograms/scissors

Different individual kinematic chains or 4dof PM (parallel) mechanisms architectures can result from the variation of the triangle B1AB2 outline height (h) and the distance between the parallel links (d). The variable parameters - (ai) is the distance between two sides (center line of each paralelograms) and can be easy related of the former ones by the simple geometrical relations in a general triangle, and/or by particulars criteria imposing to the sides, as for example to be equal (li=ln). (The process can be followed also in a opposite way, by starting from an trapezoid). Anti-parallelograms or crossed structures scissors (b=0) should be taken in to the consideration as potentially viable working structures, too. Even if their workspace are considerable reduced comparing with their counterparts (ordinary Pa) ones. Among some advantages of having individual and cooperative structures to carry specific tasks, we want to stress the possibility to escape and/or use the singular positions as working positions, suggested in several papers before, or to be escaped if the unactuated sides are to be not equal.

2-2[(PP)/RR/RR] kinematic characteristics are in the followings investigated by using adequate tools and methodology in relation with the fundamental unit (MU-0).

IV. FUNDAMENTAL MANIPULATION UNIT (MU-0)

A. Topology

There are several ways to mechanical arrange the pairs with different dof, and to link them together with rigid bodies (links) to form a kinematic chain, and to finally obtain a mechanism with a desired motion and/or force transmission for the purpose of work. PM consist from several mechanical pairs (or, joints) and links –generally more than serial ones, so formed that it can transmit the motion, and/or force by a least two actuators.

A topology analysis of a structure consist in studying the type (dof only) and number of links and pairs and their disposal (levels) in a kinematic chain (KC), and the incidence with it will appear. As the first step in any kinematic analysis of new designed mechanism, it reveals the specific potential features (kinematic/dynamic) of the mechanism, together with those general, of the entire class(family) whom it comes.

All our Manipulation Units (MUs) components of the system are centered (based) and derived from fundamental structure shown in Fig. 3a (or, b-symetric in rapport with trapezoid points). The graphs of these fundamental 4dof structures resulted from a systematic investigation and work towards to find adequate and suitable structures in accordance with several imposed criteria and requirements presented in detail in [15].

They can be seen as derived from the representative member of the fully 4dof PM family, specifically 4-4[(1111)] type, by concentrating the actuation in only two points or as symmetric one, in two points actuated 4-2[(1111)] by adding in a symmetrically way, two additionally pairs and one links, between the level III and IV –Fig.3 a, and/or II-III –Fig. 3b, respectively. The contact (support) number of points will increase to four, totally (one in addition to each chain). This is done from the kinematic reasons, in order to keep the motion of end-effector (platform) in a permanent translation, by preserving to it the possibility to be rotated in one direction. (A single link would be enough to fulfill the requirements, but from the advantages of symmetry and generality we adopted and kept this form. See next chapter). The structures can be
written as $4-2[2][1][1-1][1]$ and $4-2[2][1][1][1-][1]$ being symmetric each other.

In this form, each of the structures are symmetric and consists from only two branches (legs), each of them contains four (IV) levels going upwards from the base, and the first (I) is actuated as shown in structure schemes presented.

Only one degrees of freedom passive (unactuated) pairs are used at the levels (I-III) forming another two symmetric closed loops. As a result, the graph represent a (sub)class of Symmetric Spatial Parallel Mechanisms (SSPM) with 4dof. It includes the (general) advantages of this class as for example the minimum number of legs, etc plus others potentially (e.g. multidegree of freedom actuation, etc). One branch has more than 6dof, but with some geometric arrangements partially constrains the endeffector to move only with 4dof as will be shown. How other structures derived from it are presented next.

B. Mobility

Mobility is another important features to characterize a manipulation system (unit). It has the scope of shown the capability of motion for its mechanism, and for a PM it reflect the mobile platform degrees of freedom.

Before caring out this step, we used the well known fundamental expression – Kutzbach-Gröbler formula, in the process of systematization and finding out suitable structures for the manipulation system. This resulted in discovering some already existent parallel mechanisms structures and to propose new others. Standard, representative members of 4dof family were easy to compute their dof and by this to find their mobility level with

$$M=6(N-g)+\sum_{i=1}^{g}C_i$$  \hspace{1cm} (1)

where “N” and “g” are the total number of links and pairs in a mechanism, and $c_i$ the degrees of freedom for $i$-th pair type.

But, in this specific case, as happened with other complex parallel structures which are using especially, the complex pairs, as parallelograms, the result can not provide us a totally satisfaction. There are totally ten ($C_s=10$) low level pairs (1dof) and two ($C_s=2$) more complex pairs (2dof) in addition to ($N=9$) links. Then, with (1) the M becomes negative number (-4) which means the mechanism is over constrained. The true is hidden behind of some undetected particulars of the real structure of the mechanism, and should be seen in direct relation with the kinematic (geometric) scheme.

Firstly, by concentrating the actuation in only two (2) points (one for a kinematic chain), has as a consequence to choose normally for the mobile platform to have the same number of points (2). This is resulting in a planar mechanism which EE as a bar and constrained to move and to have maximum 3 dof from totally six (6). But, to dispose the remaining two (2) contact points (collinear) on EE is resulting in another constrained to it and by this to move only in transversal motion (reducing its dof from three (3) to two (2)). Additionally, the remaining dof for each actuator (1)can move the entire plan containing the mechanism (and, EE) in translation and rotate it along a vertical axis belonging to it. Taken in to the consideration that these structures are equivalent with, and resulted (by, adding links and pairs) at one of the standard member of 4dof SSPM family ($4-2[2][1][1][1]$) the correct result is obtained from there: $F=4$ ($N=7$, $C_s=6$, $C_d=2$). The mobility (and, dof) of the mechanism can be seen, as resulting from – a) an internal mobility, specific to the planar linkage (complex four bars linkage with only one branch parallelogram and two (collinear actuators) providing 2dof translational, plus - b) a general mobility (of the plan) related with its motion (translation along the remaining axis and the rotation around a vertical axis, belonging to it). The last one axis can pass through any the actuators points, center of the end effector or other random points, as it will be more in detail explained in the next chapters.
Making such a geometry analysis is one of the steps necessary in the kinematical analysis of a new mechanism, helping us to better understand and known the real resulted motion, and its specific capabilities (eventually constraints).

There are several possibilities to introduce in a parallel mechanism the 2dof motion. This depends on the applications specific. The actuation can be introduced from outside or inside of the base, by every kind of two dof device, but in our case we are interested to be as closed loop, too. By choosing from outside one we choose a parallel actuated (actuation) one. This restriction would be not only in the scope to maintain (or, to increase) the advantages gain by the general parallel structure (mechanism), but is only one way to solve the technical requirements for some applications, as in our case is (large workspace).

We choose the orthogonal planar pair - P_\perp P, or in a shorter symbol (2P) or (Pi). The last symbol, denote a XY planar pair which practically exists, as linear direct drive compact (planar) motor. The mobile bodies could be two identical rigid bodies blocks (but, not strict necessary). These two rigid bodies (blocks) is moving with a limited stroke on a common (or, not) flat surface (base) providing for every leg (arm) 2dof.

The platform, as a rigid body (bar) is linked by two x four identical R pairs (parallelograms) with these mobile bodies (actuators) through another two R pairs, by forming two (2) identical kinematic chains (PP)R-2R/2R.

The first R pair has its axis perpendicular on the actuator actuation plan (base) in order to be accommodate with the movement coming from the other actuator - (PP)\perp R. In the horizontal actuation case, this means it will be vertical, as in figure is shown. From the symmetry reasons the KC is to be identical. Each (vertical) axis of these pairs (BiCi) intersects at one common point (Ci) the first horizontally axes of two (RR) pairs which are in fact parallel with other ones belongs to the parallelogram situated on the next level (i=1,2).

The fundamental geometry of the mechanism derived from the general structural one, as depicted in Fig. 3a, shown in Fig. 4, is called 4-2[(PP)R-RRR], or 4-2[(PP)R-2R/2R] or more shortly 4-2[(Pi)R-4R], or 4-2[(2Pi)R-4R]. (Pi-2dof planar pair).

For the advantages of symmetry every kinematic chain (Kc) taken in to account the reciprocal location of the axis can be written as (\textbf{P}_\perp \textbf{L})R_\perp R \parallel \textbf{R} \parallel \textbf{R} or, in a more shortest form (\textbf{Pi})L.R\perp R.R. (Other possibilities can be 4-2[(\textbf{Pi})R-4R/R]).

We can consider also nonsymmetric simplified (SSPM) structural schemes of the fundamental geometry to form a mechanism, having only one link in a branch.

For those, fundamental and particular, the specific properties for these structures are coming as the intrinsic properties of the geometry - number of outer and inner chains (k=2), total elements (N=9) and joints (C_r=10, C_m=2) and the chosen values for the geometric/kinematic design parameters-link lengths (R_i), distance between links (d_i) and chains (a_i) and actuators size (l_i, l_j) or the height (h_i) of the joints of the architecture (as previously mentioned).

In accordance with the general geometric model (GM), Fig. 5, points A_i belonging to the end effector (platform), will occupy different positions in the space, when the mechanism works - curves (C_i). These mobile curves of guidance (MCG) have each 2dof and a specific shape as will be shown further.

The linear displacements in actuated pairs q_i, i,j=1,2 are the generalized coordinates of the mechanism and the generalized coordinates of the manipulated object are the Cartesian coordinates (X,Y,Z) of the point (A)-center of the platform and angle (\varphi) which will change their values in the workspace, according with the previous ones through the curvilinear coordinates, w_i (i=1,2,3). The mobile reference frame P-X'y'z' is tied to the manipulated object, and the o-xyz ones to the mobile platform.

Motion at the EE is achieved by the individual and/or concurrent movement actions of the pairs actuated who transmit the motion through the scissor (and, parallelograms). Due to the specific combination of pairs in an individual chain, and the linearity of platform points, EE will have the possibility to move only in translational motions. By the R pairs from the first level, the upper linkage can be rotated around the vertical axis each of them, or a common point between them on the line passing through.

If we build the actuators (2dof) mounted on the base as planar motors, the number of structural components, is decreasing, and can be expected from this structure a low risk of the intersections between elements in its workspace.

We can conclude that by reducing the actuated and guided number of points from four (generally) to three or four (defining a line) could offer a smaller number of the kinematical chains (three) and therefore the number of links and pairs. The proposed mechanism can work in turn with an increased workspace because not so many parts are in movement. The structure will be a serious candidate for fully PM waiting for the confirmation. (even if, the actual definition doesn’t accept it).

VI. POSITIONAL PROBLEME

The movement of every rigid body is known if its position at that moment is known. This position is materialized by various parameters allocated to a point, and or rigid body as lengths, and angles, which unequivocably represent their position. We can
use the mechanisms vector outline, or only the geometrical methods to find them.

The positional problems - direct and/or inverse, in the analysis of every new mechanisms is not only a fundamental and first steps towards the correct and well-understanding of the mechanisms work, but sometimes is a challenging task. (This is not our case.) By solving D/IPP we pave the way for a fast and correct design and control procedures.

When mechanisms are not complicated, sometimes, the fundamental and specific notions of the elementary and/or spatial geometry can provide enough

\[
\begin{align*}
X_i &= X_i(q_{i1},q_{i2},\alpha_i,\beta_i) = q_{i1} + R_i \cos \varphi_i \\
y_i &= Y_i(q_{i1},q_{i2},\alpha_i,\beta_i) = q_{i2} + R_i \sin \varphi_i \\
z_i &= Z_i(q_{i1},q_{i2},\alpha_i,\beta_i) = h_i + R_i \cos \theta_i \\
\end{align*}
\]  

where \( w_i \) and \( \varphi_i \) are auxiliary variables (angles), and \((R_i, h_i)\) constant parameters of the mechanism, links length and heights of the first parallelogram joints.

The coordinates of the same points \( A_i \) in terms of the mobile platform center \( A \) coordinates \((X, Y, Z)\) or one more general point \( P \) belonging to the manipulated object \( M \) \((X_P, Y_P, Z_P)\) tied of platform, taken in to account also the orientation, can be expressed in the same system \((O-XYZ)\), but taken in to account the coordinates of points \( A_i \) - \( (x_i, y_i, z_i) \) and \( P \) - \( (x_P, y_P, z_P) \) in the mobile frame \((O-xyz)\) are

\[
\begin{align*}
X_i &= X_i(X_P, Y_P, Z_P, \varphi) = X_P + (x_i - x_P) \alpha + (y_i - y_P) \beta \\
y_i &= Y_i(X_P, Y_P, Z_P, \varphi) = Y_P + (x_i - x_P) \beta + (y_i - y_P) \alpha \\
z_i &= Z_i(X_P, Y_P, Z_P, \varphi) = Z_P + (z_i - z_P) \gamma \\
\end{align*}
\]  
i = 1, 2, 3

where \( \alpha, \beta, \ldots, \gamma \) are the directory cosines of \( o-xyz \) frame in terms of \( O-XYZ \), one. (In our case, \( oz \parallel OZ \) during the motion)

By (1) and (2) equalization, the obtained expressions stand as positional equations for the mechanism. Their Cartesian parametric form includes \( w_i \) and \( \varphi_i \) as variable curvilinear parameters.

\[
\begin{align*}
X_i &= X_i(q_{i1},q_{i2},\varphi,\alpha_i,\beta_i,\gamma_i) = q_{i1} + R_i \cos \varphi_i + \gamma_i \\
y_i &= Y_i(q_{i1},q_{i2},\varphi,\alpha_i,\beta_i,\gamma_i) = q_{i2} + R_i \sin \varphi_i + \beta_i \\
z_i &= Z_i(q_{i1},q_{i2},\varphi,\alpha_i,\beta_i,\gamma_i) = h_i + R_i \cos \theta_i + \alpha_i \\
\end{align*}
\]

But, from the mechanism construction the following relation exists

\[
\begin{align*}
\varphi_i &= \varphi_1 + (i-1)\pi, i = 1, 2, 3 \\
\end{align*}
\]

(3) and (4) can be seen as an equation system in eight \((8)\) unknowns, which could be used to solve the positional problem. But, the problem can be further simplified, by the \( w_i \) elimination from (1)

\[
\begin{align*}
F_i &= F_i(q_{i1},q_{i2},X_i,Y_i,Z_i,\varphi) = (X_i - q_{i1})^2 + (Y_i - q_{i2})^2 + (Z_i - h_i)^2 - R_i^2 = 0 \\
G_i &= G_i(q_{i1},q_{i2},X_i,Y_i,Z_i,\varphi) = \gamma_i (Y_i - q_{i2}) + (X_i - q_{i1}) = 0, i = 1, 2 \\
\end{align*}
\]

By substituting (3) in (5) and taken in to account (4) this system containing four nonlinear equations with four unknown \((X_i,Y_i,Z_i,\varphi)\) or \((q_{i1},q_{i2},i=1,2)\) and can better help the inverse or direct problem derivation for the mechanisms positions point of view.

Geometrically, this implicit Cartesian system of equations represents two (mobile) curves-(C). This curves are symmetric circles - \( R_i \) radius (constant) with 2dof mobile center - \( D_i \) \((X_{Di},Y_{Di},Z_{Di})\) which result as an intersection between two (mobile) spheres of guidance \( S_i \) with one \( (1) \) dof and a plan \( (\pi_i) \) perpendicular on the base \( (Z_i=0) \) containing the \( B_i \) actuated points with \( \varphi_i(\phi) \) orientation.

The points \( A_i \) are forced (guided) to move and to follow a path \((\Gamma_i)\) during the entire mechanisms motion by keeping an permanent contact with these \((C_i)\) - mobile curves of guidance (MCG).

As for our manipulator \( A_i \) have the coordinates

\[
\begin{align*}
x_i &= a_i \cos \delta_i, y_i = 0, z_i = 0, \delta_i = (2-i)\pi, i = 1, 2 \\
\end{align*}
\]

we obtain from (2)
The motion in XOY plane (Z=const) around one point (axis) is given by:

\[
\begin{align*}
X_i &= X_i^0 + a_iC_i^0\varphi_i \\
Y_i &= Y_i^0 + a_iS_i^0\varphi_i \\
Z_i &= Z_i^0 \\
\end{align*}
\]

where \((X, Y, Z)\) are the Cartesian coordinates of a point on the platform (bar). By substituting (7) in (5) we obtain

\[
\begin{align*}
\sum f_i &= f_i(q_i, q_{i+1}, X, Y, Z, \varphi) = (q_{i1}^2 + (q_{i2})^2 + (A_i q_{i1} + B_i q_{i2}) + C_i = 0 \\
\sum g_i &= g_i(q_i, q_{i+1}, X, Y, Z, \varphi) = (D_i q_{i1} + E_i q_{i2}) + F_i, \quad i=1, 2 \quad (8)
\end{align*}
\]

which are the input-output system of equations for the mechanism (here expressed in terms of \(q_i, q_{i+1}\)).

As can be seen, the are two sets of solutions - one set of them \((q_{i1}, q_{i2})_1\), with "+ " in front of the radical, corresponding to the physique (real) mechanism’s position, when the points \(B_i\) (\(C_i\)) of the actuators are in the plane of the mechanism but outside of the end-effector/platform line (regarding the vertical center of the platform/bar).  By substituting (7) in (5) we obtain the solutions for the inverse positional problem (IPP) of the mechanism.

But, by using the advantage of MCG equations (5) already fund out before, the analytically expressed solutions are easier yields as

\[
\begin{align*}
\begin{cases}
(q_{i1})_{1,2} &= X+\sigma a C_i \varphi_i -tb C_i \varphi_i \\
(q_{i2})_{1,2} &= Y+\sigma a S_i \varphi_i -tb S_i \varphi_i
\end{cases} \quad i=1, 2 \quad (9)
\end{align*}
\]

where
\[
\begin{align*}
b_i &= R^2 - (Z_i - h_i)^2/2 \\
\sigma &= (-1)^i, \quad i=1, 2 \\
\tau &= \pm 1
\end{align*}
\]

As was shown from there, the modular manipulation system can be applied to for linear (translation) of the mobile platform -p, as well as the orientation of the manipulated body, we can use also:

\[
tg\varphi = (q_{i2} - q_{i1})/(q_{i2} - q_{i1}) \quad (12)
\]

And, for \(w_i\) determination

\[
w_i = (Z_i - h_i)/ R_i \quad (13)
\]

VII. OTHER MANIPULATION UNITS (MU-X)

As was previously sometimes mention here (chapter III), other symmetric manipulation units (MU-X), as Mu1, Mu2 or Mu3, are kinematic deriving from the fundamental one by design changes which does not affect the structure of the fundamental mechanism. The previous performed analysis and results from topological point of view remain valid (chapter IV-A). The exceptions occur only in the cases of (nonsymmetric) mechanism with only one link on one branch (kinematic chain), when \(F\) becomes nul (\(F=0\)). In this case \(C_4\) remain 2, but \(C_5=8\), and \(N=8\). Moreover, for more particular case when both branches have a single and only one link, as Mu4, the \(F =–2 (C_4=2, C_5=6, N= 6)\). The mechanism could be seen as with only three dof (end effector is theoretically a point) and as a result the mechanism will perform translational motions, only.

VIII. CONCLUSIONS

As was shown from there, the modular manipulation system could perform simple task involving 1 or 2 and up to 4 dof positioning consisting in moving each of the actuators by their self or, in a cooperative manner (more than one) for accomplish with the tasks in a assembly cell environment involving - alignment, transport, etc or other manufacturing/assembly operations (drilling, milling, threads, screwing, etc). Different combinations of 4 dof PM based on the fundamental manipulation unit (Mu-0) were recommended, and analyzed to tailor the performance for a given application with multiple requirements of manipulation capabilities.

This kinematics analysis can stand as a first step in to design a successful and improved further PM system.
REFERENCES


